New Charm Spectroscopy: Insights from Theory

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Abstract

We discuss several new observations of mesons with open charm. In particular, we consider $D_{sJ}(2317)$ and $D_{sJ}(2460)$ and compare their isospin violating decays into $D_s^{(*)}\pi$ to the radiative decays analysed using light-cone QCD sum rules. The results support the interpretation of these two mesons as ordinary $c\bar{s}$ states.

In the case of $D_{sJ}(2860)$ and $D_{sJ}(2710)$ we compute the strong decays in the heavy quark limit. Comparison of the results with recent measurements of the BaBar Collaboration leads to identify $D_{sJ}(2710)$ with the first radial excitation of D_s^*, while the identification is still uncertain in the case of $D_{sJ}(2860)$.

Introduction

Starting from 2003, charm spectroscopy has entered a new era, due to a series of intriguing observations both in the open and in the hidden charm sectors [1]. Some of the newly observed states can be easily classified within the quark model scheme, some others still wait for a proper identification. Here we focus on mesons with charm and strangeness, in particular on the narrow states $D_{sJ}(2317)$ and $D_{sJ}(2460)$, observed in 2003, and on the states $D_{sJ}(2860)$ and $D_{sJ}(2710)$, discovered in 2006 and in 2007, respectively. As a preliminary step, we introduce a suitable classification of mesons with a single heavy quark which can be derived in the infinite heavy quark mass limit, exploiting the symmetries of QCD in such a limit.

Hadrons with a Single Heavy Quark

In the infinite heavy quark mass limit, $m_Q \to \infty$, the QCD Lagrangian is invariant under heavy quark spin and flavour rotations, and an effective theory can be built, known as Heavy Quark Effective Theory (HQET) [2].

Let us consider hadrons with a single heavy quark Q. When $m_Q \to \infty$, Q acts as a static colour source for the light degrees of freedom (*ldf*) of the hadron. In the case of mesons, considered here, *ldf* consist of the light antiquark $\bar{q}$ and gluons. In particular, the heavy quark spin s_Q is no more coupled to the *ldf* total angular momentum s_ℓ, given by $\vec{s}_\ell = \vec{s}_q + \vec{\ell}$, where s_q is the light antiquark spin and ℓ its orbital angular momentum with respect to Q. Therefore, a heavy hadron can be labelled not only according to its total spin $\vec{J} = \vec{s}_Q + \vec{s}_\ell$, but also to the value of s_ℓ. An important consequence is that states which differ only for the orientation of s_Q with respect to s_ℓ are expected to be degenerate, and this allows to collect heavy mesons in doublets, the members of which have the same value of s_ℓ and correspond to the two possible orientations of s_Q with respect to s_ℓ. Finite heavy quark mass corrections remove the degeneracy between the members of a doublet and induce a mixing between states with the same spin-parity J^P belonging to different doublets.

Let us consider the lowest doublets that can be built according to this classification. For $\ell = 0$ one has a doublet of states (P, P^*) with $J^P_{s_\ell} = (0^-, 1^-)_{1/2}$ (we refer to this as to the *fundamental* doublet), while two doublets correspond to $\ell = 1$: (P_0^*, P_1') with $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$ and (P_1, P_2^*) with $J^P_{s_\ell} = (1^+, 2^+)_{3/2}$. For the purposes of this paper, we need to introduce also the two doublets corresponding to $\ell = 2$: $J^P_{s_\ell} = (1^-, 2^-)_{3/2}$ and $J^P_{s_\ell} = (2^-, 3^-)_{5/2}$. For each doublet, one can consider a tower of states corresponding to the radial excitations.

In the heavy quark limit one can predict whether these states are narrow or broad. For example, strong decays of the states belonging to the $J^P_{s_\ell} = (1^+, 2^+)_{3/2}$ doublet to the fundamental doublet with the emission of a light pseudoscalar meson occur in d-wave. Since the rate for this process is proportional to $|\vec{p}|^5$ (in general, to $|\vec{p}|^{2\ell+1}$, p being the light pseudoscalar momentum and l the angular momentum transferred in the decay), these states are expected to be narrow. On the contrary, states belonging to the $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$ doublet decay in s-wave, hence they should be broad.

Experimental data collected up to 2003 show that heavy mesons fit very well in this scheme, as can be argued looking at the experimental values of masses and widths: the degeneracy condition is better fulfilled by beauty mesons than by charmed ones, as should be, the b quark being approximately three times heavier than charm. As for the widths, in Table 1 we collect masses and widths of the $c\bar{u}$ states identified as the members of the $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$ and $J^P_{s_\ell} = (1^+, 2^+)_{3/2}$ doublets [3]. From this table one can see that the members of the $J^P_{s_\ell} = (1^+, 2^+)_{3/2}$ doublet are indeed narrow, while the widths of the analogous states belonging to the $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$ doublet are much broader.

In the case of mesons with charm and strangeness, known states are those composing the fundamental doublet: $D_s(1968)$ and $D_s^*(2112)$ and the two mesons which can be assigned to the $J^P_{s_\ell} = (1^+, 2^+)_{3/2}$ doublet: $D_{s1}(2536)$, whose width is < 2.3 MeV, and $D^*_{s2}(2573)$ with measured width: $\Gamma(D^*_{s2}) = 20 \pm 5$ MeV [3]. In the following, we analyse the other mesons with charm and strangeness recently observed.

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Table 1: Masses and widths of $c\bar{u}$ states belonging to the $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$ and $J^P_{s_\ell} = (1^+, 2^+)_{3/2}$ doublets.

s_ℓ^P	J^P	state	M (MeV)	Γ (MeV)
$\frac{1}{2}^+$	0^+	D_0	2400	$283 \pm 24 \pm 34$
	1^+	D_1'	2430	$384 \pm^{107}_{75} \pm 74$
$\frac{3}{2}^+$	1^+	D_1	2420	20.4 ± 1.7
	2^+	D_2^*	2460	43 ± 4

$D_{sJ}(2317)$ and $D_{sJ}(2460)$

In April 2003 the BaBar Collaboration reported the observation of a narrow peak in the $D_s\pi^0$ invariant mass distribution with mass close to 2.32 GeV and width consistent with the experimental resolution [4]. The resonance, named $D_{sJ}(2317)$, was observed in both the $\phi\pi^+$ and $\overline{K}^{*0}K^+$ decay modes of D_s^+. The peak was also found by reconstructing D_s through $D_s \to K^+K^-\pi^+\pi^0$. No evidence for $D_{sJ}(2317) \to D_s\gamma, D_s^*\gamma$ and $D_s\gamma\gamma$ was found. The observation was confirmed by Belle [5], CLEO [6] and Focus Collaboration [7].

The decay $D_{sJ}(2317) \to D_s\pi^0$ implies for $D_{sJ}(2317)$ natural spin-parity. The helicity angle distribution of $D_s\pi^0$ obtained by BaBar is consistent with the spin 0 assignment, even though it does not rule out other possibilities; the absence of a peak in the $D_s\gamma$ final state supports the spin-parity assignment $J^P = 0^+$. The measured mass is below the DK threshold $\mathrm{M}_{D^+K^0} = 2.36$ GeV.

Together with the $D_{sJ}(2317)$, CLEO Collaboration reported the observation of a narrow resonance, $D_{sJ}(2460)$, in the $D_s^*\pi^0$ system [6], with mass close to 2.46 GeV and width consistent with the experimental resolution. Later on, also radiative decays of $D_{sJ}(2460)$ have been detected, with measured branching fractions: $BR(D_{sJ}(2460) \to D_s\gamma) = (18 \pm 4)\,10^{-2}$ and $BR(D_{sJ}(2460) \to D_{sJ}(2317)\gamma) = (3.7\pm^{5.0}_{2.4})\,10^{-2}$, while the upper limit $BR(D_{sJ}(2460) \to D_s^*\gamma) < 8\%$ [3] was put. Angular analyses suggest the assignment $J = 1$. The mass of $D_{sJ}(2460)$ is below the D^*K threshold $\mathrm{M}_{D^{*+}K^0} = 2.51$ GeV.

Being two states with $J^P = (0^+, 1^+)$ their natural interpretation would be as the components of the doublet with $s_\ell^P = \frac{1}{2}^+$. However, this interpretation raises several questions. The first one stems from the comparison with potential model predictions of the masses, which correspond to larger values, above the threshold allowing isospin conserving decays (DK and D^*K in the two cases). The second one is that the members of the $s_\ell^P = \frac{1}{2}^+$ doublet are expected to be broad, while the observed mesons are narrow. Many interpretations have been provided since the original discovery of these states [1]. However there are arguments to support the interpretation of $D_{sJ}(2317)$ and $D_{sJ}(2460)$ as ordinary $c\bar{s}$ states, their narrowness being due to the low mass forbidding isospin conserving decays.

An example of such arguments is based on the analysis of radiative transitions, that probe the structure of hadrons [8, 9]. Identifying $D_{sJ}(2317)$ with D_{s0}^* and $D_{sJ}(2460)$ with D_{s1}', the decay amplitudes governing the $D_{s0}^* \to D_s^*\gamma$ and $D_{s1}' \to D_s^{(*)}\gamma$, $D_{s0}^*\gamma$ transitions:

$$\begin{aligned}
&\langle\gamma(q,\lambda)D_s^*(p,\lambda')|D_{s0}^*(p+q)\rangle \\
&= e\,d\,[(\varepsilon^*\cdot\tilde{\eta}^*)(p\cdot q)-(\varepsilon^*\cdot p)(\tilde{\eta}^*\cdot q)] \\[1em]
&\langle\gamma(q,\lambda)D_s(p)|D_{s1}'(p+q,\lambda'')\rangle \\
&= e\,g_1\,[(\varepsilon^*\cdot\eta)(p\cdot q)-(\varepsilon^*\cdot p)(\eta\cdot q)] \qquad (1) \\[1em]
&\langle\gamma(q,\lambda)D_s^*(p,\lambda')|D_{s1}'(p+q,\lambda'')\rangle \\
&= i\,e\,g_2\,\varepsilon_{\alpha\beta\sigma\tau}\eta^\alpha\tilde{\eta}^{*\beta}\varepsilon^{*\sigma}q^\tau \\[1em]
&\langle\gamma(q,\lambda)D_{s0}^*(p)|D_{s1}'(p+q,\lambda'')\rangle \\
&= i\,e\,g_3\,\varepsilon_{\alpha\beta\sigma\tau}\varepsilon^{*\alpha}\eta^\beta p^\sigma q^\tau
\end{aligned}$$

involve the the hadronic parameters d, g_1, g_2 and g_3 ($\varepsilon(\lambda)$ is the photon polarization vector and $\tilde{\eta}(\lambda')$, $\eta(\lambda'')$ the D_s^* and D_{s1}' polarization vectors). Such parameters can be computed by light-cone sum rules [10]. Considering the correlation functions [11, 12]

$$F(p,q) = i\int d^4x\; e^{ip\cdot x}\langle\gamma(q,\lambda)|T[J_A^\dagger(x)J_B(0)]|0\rangle \quad (2)$$

of quark-antiquark currents $J_{A,B}$ having the same quantum number of the decaying and of the produced charmed mesons, and an external photon state of momentum q and helicity λ, and expanding on the light-cone, it is possible to express F in terms of the perturbative photon coupling to the strange and charm quarks, together with the contributions of the photon emission from the soft s quark, expressed as photon matrix elements of increasing twist [13], see fig.1 . The hadronic representation of the correlation function involves the contribution of the lowest-lying resonances, the current-vacuum matrix elements of which are computed by the same method [14], and a continuum of states treated invoking global quark-hadron duality. The final step of the method consists in applying to both the representations of the correlation function a Borel transformation, which improves in several respects the sum rule while introducing an external parameter M^2. The hadronic quantities should be independent of it, so that the final results are found requiring stability against variations of M^2 (fig. 2).

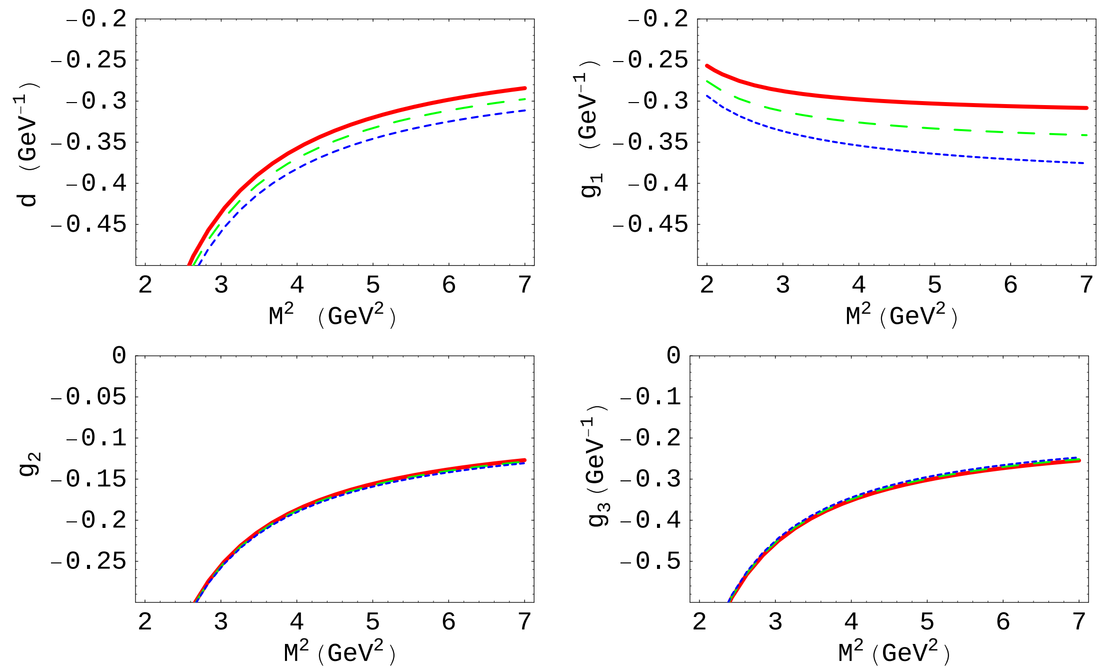

Figure 2: Light-cone sum rule results for the hadronic parameters governing radiative decays of $D_{sJ}(2460)$ and $D_{sJ}(2317)$; M^2 is the Borel parameter.

Table 2: Radiative decay widths (in keV) of $D_{sJ}(2317)$ and $D_{sJ}(2460)$ obtained by Light-Cone sum rules (LCSR), Vector Meson Dominance (VMD) and constituent quark model (QM).

Initial state	Final state	LCSR [10]	VMD [9]	QM [8]	QM [15]
$D_{sJ}(2317)$	$D_s^*\gamma$	4-6	0.85	1.9	1.74
$D_{sJ}(2460)$	$D_s\gamma$	19-29	3.3	6.2	5.08
	$D_s^*\gamma$	0.6-1.1	1.5	5.5	4.66
	$D_{sJ}(2317)\gamma$	0.5-0.8	—	0.012	2.74

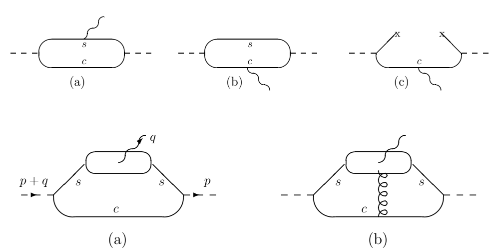

Figure 1: Leading contributions to the correlation functions eq. (2) expanded on the light-cone: perturbative photon emission by the strange and charm quark ((a,b) in the first line) and two- and three-particle photon distribution amplitudes (second line); (c) corresponds to the strange quark condensate contribution.

In Table 2 the light-cone QCD sum rule results are collected together with the results of other methods [9, 8, 15]. Looking at this Table one can see that the rate of $D_{sJ}(2460) \to D_s\gamma$ is the largest one among the radiative $D_{sJ}(2460)$ rates, and this is confirmed by experiment [3]. Quantitative understanding of the experimental data for both hadronic and radiative decays requires a precise knowledge of the widths of the isospin violating transitions $D_{s0}^* \to D_s\pi^0$ and $D'_{s1} \to D_s^*\pi^0$. In the description of these transitions based on the mechanism of $\eta - \pi^0$ mixing [8, 9] the accurate determination of the strong $D_{s0}^* D_s \eta$ and $D'_{s1} D_s^* \eta$ couplings for finite heavy quark mass and including $SU(3)$ corrections is required. These results suggest the identification of $D_{sJ}(2317)$ and $D_{sJ}(2460)$ as the two members of the $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$ doublet. Together with $D_{s1}(2536)$ and $D_{s2}^*(2573)$, these two states fill the four p-wave levels. In principle, the two $J^P = 1^+$ mesons $D_{s1}(2536)$ and $D_{sJ}(2460)$ could be a mixing of the $s_\ell^P = \frac{1}{2}^+$ and $s_\ell^P = \frac{3}{2}^+$ states, allowed at $\mathcal{O}(1/m_Q)$, an issue discussed in the last section.

$D_{sJ}(2860)$ and $D_{sJ}(2710)$

In 2006, BaBar observed another $c\bar{s}$ meson, $D_{sJ}(2860)$, decaying to D^0K^+ and D^+K_S, with mass and width [16]:

$$\begin{aligned} M(D_{sJ}(2860)) &= 2856.6 \pm 1.5 \pm 5.0 \text{ MeV} \\ \Gamma(D_{sJ}(2860)) &= 47 \pm 7 \pm 10 \text{ MeV} . \end{aligned} \quad (3)$$

Shortly after, analysing the D^0K^+ invariant mass distribution in $B^+ \to \bar{D}^0 D^0 K^+$ Belle Collaboration established the presence of a $J^P = 1^-$ resonance, $D_{sJ}(2710)$, with [17]:

$$\begin{aligned} M(D_{sJ}(2710)) &= 2708 \pm 9^{+11}_{-10} \text{ MeV} \\ \Gamma(D_{sJ}(2710)) &= 108 \pm 23^{+36}_{-31} \text{ MeV} . \end{aligned} \quad (4)$$

To classify $D_{sJ}(2860)$ and $D_{sJ}(2710)$, one can analyse the strong decays, comparing the predictions which follow from different quantum number assignments. This can be done using an effective Lagrangian approach which exploits the symmetries that Quantum Chromodynamics (QCD) exhibits in specific limits. One is chiral $SU(N_f)_L \times SU(N_f)_R$ symmetry holding in the limit of N_f massless quarks. This symmetry is spontaneously broken to $SU(N_f)_V$ and light pseudoscalar mesons are identified as Goldstone bosons acquiring mass when explicit symmetry breaking mass terms are considered. An effective theory, chiral perturbation theory, can be built as an expansion in the light quark masses and momenta [18]. The other one is the heavy quark spin-flavour symmetry for $m_Q \to \infty$.

Interactions of heavy mesons with light ones can be described by an effective Lagrangian displaying both heavy quark and chiral symmetry. The Lagrangian was first formulated in the case of light pseudoscalars [19], and extended to include light vector mesons [20].

In the heavy quark limit, the doublets defined in the previous Sections are described by effective fields: H_a for $s_\ell^P = \frac{1}{2}^-$ ($a = u, d, s$ is a light flavour index), S_a and T_a for $s_\ell^P = \frac{1}{2}^+$ and $s_\ell^P = \frac{3}{2}^+$, respectively; X_a and X'_a for $s_\ell^P = \frac{3}{2}^-$ and $s_\ell^P = \frac{5}{2}^-$, respectively:

$$\begin{aligned}
H_a &= \frac{1+\not v}{2}[P^*_{a\mu}\gamma^\mu - P_a\gamma_5] \\
S_a &= \frac{1+\not v}{2}\left[P'^{\mu}_{1a}\gamma_\mu\gamma_5 - P^*_{0a}\right] \\
T^\mu_a &= \frac{1+\not v}{2}\Bigg\{P^{\mu\nu}_{2a}\gamma_\nu \\
&\quad -P_{1a\nu}\sqrt{\frac{3}{2}}\gamma_5\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\right]\Bigg\} \\
X^\mu_a &= \frac{1+\not v}{2}\Bigg\{P^{*\mu\nu}_{2a}\gamma_5\gamma_\nu \\
&\quad -P^{*\prime}_{1a\nu}\sqrt{\frac{3}{2}}\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\right]\Bigg\} \\
X'^{\mu\nu}_a &= \frac{1+\not v}{2}\Bigg\{P^{\mu\nu\sigma}_{3a}\gamma_\sigma - P^{*'\alpha\beta}_{2a}\sqrt{\frac{5}{3}}\gamma_5\Bigg[g^\mu_\alpha g^\nu_\beta \\
&\quad -\frac{1}{5}\gamma_\alpha g^\nu_\beta(\gamma^\mu - v^\mu) - \frac{1}{5}\gamma_\beta g^\mu_\alpha(\gamma^\nu - v^\nu)\Bigg]\Bigg\}
\end{aligned} \quad (5)$$

with the various operators annihilating mesons of four-velocity v (conserved in strong interactions) and containing a factor $\sqrt{m_P}$. Light pseudoscalars are introduced using $\xi = e^{\frac{i\mathcal{M}}{f_\pi}}$,with:

$$\mathcal{M} = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

($f_\pi = 132$ MeV). At the leading order in the heavy quark mass and light meson momentum expansion the decays $F \to HM$ ($F = H, S, T, X, X'$ and M a light pseudoscalar meson) can be described by the Lagrangian interaction terms (invariant under chiral and heavy-quark spin-flavour transformations) [19, 20]:

$$\begin{aligned}
\mathcal{L}_H &= g\, Tr[\bar{H}_a H_b \gamma_\mu\gamma_5 \mathcal{A}^\mu_{ba}] \\
\mathcal{L}_S &= h\, Tr[\bar{H}_a S_b \gamma_\mu\gamma_5 \mathcal{A}^\mu_{ba}] + h.c. , \qquad (6) \\
\mathcal{L}_T &= \frac{h'}{\Lambda_\chi} Tr[\bar{H}_a T^\mu_b (iD_\mu \not{\mathcal{A}} + i \not{D} \mathcal{A}_\mu)_{ba}\gamma_5] + h.c. \\
\mathcal{L}_X &= \frac{k'}{\Lambda_\chi} Tr[\bar{H}_a X^\mu_b (iD_\mu \not{\mathcal{A}} + i \not{D} \mathcal{A}_\mu)_{ba}\gamma_5] + h.c. \\
\mathcal{L}_{X'} &= \frac{1}{{\Lambda_\chi}^2} Tr[\bar{H}_a X'^{\mu\nu}_b [k_1 \{D_\mu, D_\nu\}\mathcal{A}_\lambda \\
&\quad + k_2 (D_\mu D_\nu \mathcal{A}_\lambda + D_\nu D_\lambda \mathcal{A}_\mu)]_{ba}\gamma^\lambda\gamma_5] + h.c.
\end{aligned}$$

where $D_{\mu ba} = -\delta_{ba}\partial_\mu + \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)_{ba}$, $\mathcal{A}_{\mu ba} = \frac{i}{2}\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right)_{ba}$. Λ_χ is the chiral symmetry-breaking scale; we use $\Lambda_\chi = 1$ GeV. $\mathcal{L}_S$ and $\mathcal{L}_T$ describe transitions of positive parity heavy mesons with the emission of light pseudoscalar mesons in $s-$ and $d-$ wave, respectively, g, h and h' representing effective coupling constants. On the other hand, $\mathcal{L}_X$ and $\mathcal{L}_{X'}$ describe the transitions of higher mass mesons of negative parity with the emission of light pseudoscalar mesons in $p-$ and $f-$ wave with coupling constants k', k_1 and k_2. We only consider these terms: the light meson momenta involved in the $D_{sJ}(2860)$ decays are $q_K = 0.59$ GeV for D^*K final state and $q_K = 0.7$ GeV for DK final state, so it is possible that other terms in the light-meson momentum expansion, involving other structures and couplings, should be taken into account in the interaction Lagrangian. At present these terms are unknown.

At the same order in the expansion in the light meson momentum, the structure of the Lagrangian terms for radial excitations of the H, S and T doublets does not change, since it is only dictated by the spin-flavour and chiral symmetries, but the coupling constants g, h and h' have to be substituted by $\tilde{g}, \tilde{h}$ and $\tilde{h}'$. The advantage of this formulation is that meson transitions into final states obtained by $SU(3)$ and heavy quark spin rotations can be related in a straightforward way.

Let us start with $D_{sJ}(2860)$. A new $c\bar{s}$ meson decaying to DK can be either the $J^P = 1^-$ state of the $s_\ell^P = \frac{3}{2}^-$ doublet, or the $J^P = 3^-$ state of the $s_\ell^P = \frac{5}{2}^-$ one, in both cases with lowest radial quantum number. Otherwise $D_{sJ}(2860)$ could be a radial excitation of already observed $c\bar{s}$ mesons: the first radial excitation of D_s^* ($J^P = 1^-$

Table 3: Predicted ratios R_1 and R_2 (see text for definitions) for the various assignment of quantum numbers to $D_{sJ}(2860)$ and $D_{sJ}(2710)$.

$D_{sJ}(2860)$	R_1	R_2
$s_\ell^p = \frac{1}{2}^-$, $J^P = 1^-$, $n = 2$	1.23	0.27
$s_\ell^p = \frac{1}{2}^+$, $J^P = 0^+$, $n = 2$	0	0.34
$s_\ell^p = \frac{3}{2}^+$, $J^P = 2^+$, $n = 2$	0.63	0.19
$s_\ell^p = \frac{3}{2}^-$, $J^P = 1^-$, $n = 1$	0.06	0.23
$s_\ell^p = \frac{5}{2}^-$, $J^P = 3^-$, $n = 1$	0.39	0.13
$D_{sJ}(2710)$	R_1	R_2
$s_\ell^p = \frac{1}{2}^-$, $J^P = 1^-$, $n = 2$	0.91	0.20
$s_\ell^p = \frac{3}{2}^-$, $J^P = 1^-$, $n = 1$	0.043	0.163

$s_\ell^P = \frac{1}{2}^-$) or of $D_{sJ}(2317)$ ($J^P = 0^+$ $s_\ell^P = \frac{1}{2}^+$) or of $D^*_{s2}(2573)$ ($J^P = 2^+$ $s_\ell^P = \frac{3}{2}^+$).

As for $D_{sJ}(2710)$, two possibilities can be considered, since the spin is known:

- $D_{sJ}(2710)$ belongs to the $s_\ell^P = \frac{1}{2}^-$ doublet and is the first radial excitation ($D_s^{*\prime}$);
- $D_{sJ}(2710)$ is the low lying state with $s_\ell^P = \frac{3}{2}^-$ (D^*_{s1}).

In [21] and [22] we investigated the decay modes of $D_{sJ}(2860)$ and $D_{sJ}(2710)$ according to the various possible assignments with the aim of discriminating among them. The results are collected in Table 3, where we report the ratios

$$
\begin{aligned}
R_1 &= \frac{\Gamma(D_{sJ} \to D^* K)}{\Gamma(D_{sJ} \to DK)} \\
R_2 &= \frac{\Gamma(D_{sJ} \to D_s \eta)}{\Gamma(D_{sJ} \to DK)}
\end{aligned}
\tag{7}
$$

(with $D^{(*)}K = D^{(*)+}K_S + D^{(*)0}K^+$) obtained for various quantum number assignments to $D_{sJ}(2860)$ and $D_{sJ}(2710)$ on the basis of eqs. (5) and (6). Also the decay to $D_s^*\eta$ is allowed, but it is suppressed due to the limited phase space available. The ratios do not depend on the coupling constants, but only on the quantum numbers. We first discuss the entries in Table 3 which concern $D_{sJ}(2860)$.

The case $s_\ell^p = \frac{3}{2}^-$, $J^P = 1^-$, $n = 1$ can be excluded since, using the relevant term in (6) and $k' \simeq h' \simeq 0.45 \pm 0.05$ (as the h' was determined in [23]), would give $\Gamma(D_{sJ} \to DK) \simeq 1.5$ GeV, a result incompatible with the measured width.

In the assignment $s_\ell^p = \frac{1}{2}^+$, $J^P = 0^+$, $n = 2$ the decay to D^*K is forbidden. However, if $D_{sJ}(2860)$ is a scalar radial excitation, it should have a spin partner with $J^P = 1^+$ ($s_\ell^p = \frac{1}{2}^+$, $n = 2$) decaying to D^*K with a small width, a rather easy signal to detect. For $n = 1$ both $D_{sJ}(2317)$ and $D_{sJ}(2460)$ are produced in charm continuum at e^+e^- factories. To explain the absence of the D^*K in charm continuum events at mass around 2860 MeV, one should invoke some mechanism favoring the production of the 0^+ $n = 2$ state and inhibiting the production of 1^+ $n = 2$ state, a mechanism which discriminates the first radial excitation from the low lying state $n = 1$. Such a mechanism is difficult to imagine [24].

Among the remaining possibilities, the assignment $s_\ell^p = \frac{5}{2}^-$, $J^P = 3^-$, $n = 1$ seems the most likely one. In fact, in this case the small DK width is due to the suppression related to the kaon momentum factor: $\Gamma(D_{sJ} \to DK) \propto q_K^7$. The spin partner would be D^*_{s2}, the $s_\ell^P = \frac{5}{2}^-$, $J^P = 2^-$ state, which can decay to D^*K and not to DK. It would also be narrow but only in the $m_Q \to \infty$ limit, where the transition $D^*_{s2} \to D^*K$ occurs in f-wave. As an effect of $1/m_Q$ corrections this decay can occur in p-wave, so that D^*_{s2} could be broader; therefore, it is not necessary to invoke a mechanism inhibiting the production of this state with respect to $J^P = 3^-$. If $D_{sJ}(2860)$ has $J^P = 3^-$, it is not expected to be produced in non leptonic B decays such as $B \to DD_{sJ}(2860)$: the non leptonic amplitude in the factorization approximation vanishes since the vacuum matrix element of the weak $V - A$ current with a spin three particle is zero. Therefore, the quantum number assignment can be confirmed by studies of D_{sJ} production in B transitions. Actually, in the Dalitz plot analysis of $B^+ \to \bar{D}^0 D^0 K^+$ Belle Collaboration [17] has found no signal of $D_{sJ}(2860)$.

The conclusion is that $D_{sJ}(2860)$ is likely a $J^P = 3^-$ state, a predicted high mass and relatively narrow $c\bar{s}$ state [25]. Its non-strange partner D_3, if the mass splitting $M_{D_{sJ}(2860)} - M_{D_3}$ is of the order of the strange quark mass, is also expected to be narrow: $\Gamma(D_3^+ \to D^0\pi^+) \simeq 37$ MeV. It can be produced in semileptonic as well as in non leptonic B decays, such as $\bar{B}^0 \to D_3^+ \ell^- \bar{\nu}_\ell$ and $\bar{B}^0 \to D_3^+ \pi^-$ [25]: its observation could be used to assign the proper quantum numbers to the resonance $D_{sJ}(2860)$ found by BaBar. Before considering the D^*K mode, let us look at $D_{sJ}(2710)$. As Table 3 shows, R_1 is very different if $D_{sJ}(2710)$ is $D_s^{*\prime}$ or D^*_{s1}: the D^*K mode is the main signal to be investigated in order to distinguish between the two possible assignments. From the computed widths, assuming that $\Gamma(D_{sJ}(2710))$ is saturated by modes with a heavy meson and a light pseudoscalar meson in the final state, we can determine the couplings $\tilde{g}$ and k' governing the decays in the two cases. Identifying $D_{sJ}(2710)$ with $D_s^{*\prime}$ we obtain:

$$\tilde{g} = 0.26 \pm 0.05 \ , \tag{8}$$

while if $D_{sJ}(2710)$ is D^*_{s1} we get

$$k' = 0.14 \pm 0.03 \ . \tag{9}$$

These values are similar to those obtained for analogous couplings appearing in the effective heavy quark chiral Lagrangians [11, 14].

The results for $\tilde{g}$ and k' can provide information about the spin partner of $D_{sJ}(2710)$, i.e. the state belonging to the same s_ℓ^P doublet from which $D_{sJ}(2710)$ differs only

for the total spin. The partner of $D_s^{*\prime}$ ($s_\ell^P = \frac{1}{2}^-$) has $J^P = 0^-$; it is denoted D'_s, the first radial excitation of D_s, while the partner of D^*_{s1} ($s_\ell^P = \frac{3}{2}^-$) is the state D^*_{s2} with $J^P = 2^-$. In both cases, the decay modes to $D^{*0}K^+$, $D^{*+}K^0_{S(L)}$, $D^*_s\eta$, are permitted. In the heavy quark limit, these partners are degenerate. Using the obtained values for $\tilde{g}$ and k', we get:

$$\Gamma(D'_s) = (70 \pm 30) \text{ MeV} , \qquad (10)$$

and

$$\Gamma(D^*_{s2}) = (12 \pm 5) \text{ MeV} , \qquad (11)$$

so that in the two assignments the spin partners differ for their total width.

Along the same lines, one can study the charmed mesons with the same quantum numbers as $D_{sJ}(2700)$, but with a different light quark flavour. These states are a charged charmed meson and a neutral one, denoted as D^+_J and D^0_J, respectively. They have not been observed yet, so that their masses are unknown. We assume such masses to be 2600 ± 50 MeV by the reasonable assumptioncriterion that $D_{sJ}(2700)$ is heavier by an amount of the size of the strange quark mass.

Allowed decay modes for $D^+_J(2600)$ are: $D^+_J \to D^0\pi^+$, $D^+\pi^0$, $D_s\bar{K}^0_{S(L)}$, $D^+\eta$, and $D^+_J \to D^{*0}\pi^+$, $D^{*+}\pi^0$, $D^{*+}\eta$, while for D^0_J they are: $D^0_J \to D^+\pi^-$, $D^0\pi^0$, D_sK^-, $D^0\eta$ and $D^0_J \to D^{*0}\pi^0$, $D^{*+}\pi^-$, $D^{*0}\eta$; the corresponding widths depend on the possible identification of $D^{+(0)}_J$. The states having $s_\ell = \frac{1}{2}^-$ are denoted as $D^{*\prime +(0)}$ and are radial excitations, while the states having $s_\ell = \frac{3}{2}^-$ are denoted as $D^{*+(0)}_1$.

Using the effective coupling constants $\tilde{g}$ and k' in (8), (9), we obtain:

$$\Gamma(D^{*\prime +(0)}) = (128 \pm 61) \text{ MeV} \qquad (12)$$

$$\Gamma(D^{*+(0)}_1) = (85 \pm 46) \text{ MeV} \qquad (13)$$

so that the $c\bar{q}$ partners have widths which are different in the case of the two assignments. The mesons are not very broad, hence it should be possible to observe them.

We conclude this discussion mentioning that a new experimental analysis of DK and D^*K final states has been performed by BaBar Collaboration [26].

As it emerged above, the D^*K mode plays an important role in this context. BaBar has observed both $D_{sJ}(2710)$ and $D_{sJ}(2860)$ decaying to DK and D^*K final states, hence the states should have natural parity $J^P = 1^-, 2^+, 3^-, \ldots$. The assignment $J^P = 0^+$ for $D_{sJ}(2860)$ is excluded. More information comes from the measurement of the ratios [26]:

$$\frac{BR(D_{sJ}(2710) \to D^*K)}{BR(D_{sJ}(2710) \to DK)} = 0.91 \pm 0.13_{stat} \pm 0.12_{syst}$$
$$\frac{BR(D_{sJ}(2860) \to D^*K)}{BR(D_{sJ}(2860) \to DK)} = 1.10 \pm 0.15_{stat} \pm 0.19_{syst} .$$

Comparing these data with the predictions in Table 3, one concludes that

- $D_{sJ}(2710)$ is most likely $D_s^{*\prime}$, i.e. the first radial excitation of $D^*_s(2112)$;
- the ratio involving $D_{sJ}(2860)$ decays differs from the prediction at the level of three standard deviations. The identification of this state still requires further theoretical and experimental study both aiming at estimating the accuracy of the predictions in table 3 and of the experiments [27].

The last remark concerns the BaBar observation of another $c\bar{s}$ broad structure, with [26]:

$$\begin{aligned} M &= 3044 \pm 8_{stat}(^{+30}_{-5})_{syst} \text{ MeV} \\ \Gamma &= 239 \pm 35_{stat}(^{+46}_{-42})_{syst} \text{ MeV} . \end{aligned}$$

Studies of angular distributions for this state have not been attempted at present, due to the limited statistics. The theoretical analysis of this state will be reported elsewhere.

Symmetry breaking terms

Heavy quark symmetries, holding in the infinite heavy quark mass limit, are broken by terms which are suppressed by increasing powers of m_Q^{-1} [28]. Mass degeneracy between the members of the meson doublets is broken by the terms:

$$\begin{aligned} \mathcal{L}_{1/m_Q} = \frac{1}{2m_Q} \quad \cdot \quad & \{\lambda_H Tr[\bar{H}_a\sigma^{\mu\nu}H_a\sigma_{\mu\nu}] \\ & -\lambda_S Tr[\bar{S}_a\sigma^{\mu\nu}S_a\sigma_{\mu\nu}] \\ & +\lambda_T Tr[\bar{T}^\alpha_a\sigma^{\mu\nu}T^\alpha_a\sigma_{\mu\nu}]\} \end{aligned} \qquad (14)$$

where the constants λ_H, λ_S and λ_T are related to the hyperfine mass splittings:

$$\begin{aligned} \lambda_H &= \frac{1}{8}\left(M^2_{P^*} - M^2_P\right) \\ \lambda_S &= \frac{1}{8}\left(M^2_{P'_1} - M^2_{P^*_0}\right) \\ \lambda_T &= \frac{3}{8}\left(M^2_{P^*_2} - M^2_{P_1}\right) \quad . \end{aligned} \qquad (15)$$

Other two effects related to spin symmetry-breaking terms concern the possibility that the members of the $s_\ell = \frac{3}{2}^+$ doublet can decay in S wave into the lowest lying heavy mesons and pseudoscalars, and that a mixing may be induced between the two 1^+ states belonging to the two positive parity doublets with different s_ℓ. The corresponding terms in the effective Lagrangian are:

$$\begin{aligned} \mathcal{L}_{D_1} = \frac{f}{2m_Q\Lambda_\chi} \quad & Tr[\bar{H}_a\sigma^{\mu\nu}T^\alpha_b\sigma_{\mu\nu}\gamma^\theta\gamma_5 \\ & (iD_\alpha\mathcal{A}_\theta + iD_\theta\mathcal{A}_\alpha)_{ba}] + h.c. \end{aligned} \qquad (16)$$

$$\mathcal{L}_{mix} = \frac{b_1}{2m_Q} Tr[\bar{S}_a\sigma^{\mu\nu}T_{\mu a}\sigma_{\nu\alpha}v^\alpha] + h.c. \qquad (17)$$

Notice that $\mathcal{L}_{D_1}$ describes both S and D wave decays. The mixing angle between the two 1^+ states:

$$\left|P_1^{phys}\right\rangle = \cos\theta\,|P_1\rangle + \sin\theta\,|P_1'\rangle \tag{18}$$

$$\left|P_1^{\prime phys}\right\rangle = -\sin\theta\,|P_1\rangle + \cos\theta\,|P_1'\rangle \tag{19}$$

can be related to the coupling constant b_1 and to the mass splitting:

$$\tan\theta = \frac{\sqrt{\delta^2+\delta_g^2}-\delta}{\delta_g} \tag{20}$$

where $\delta = \dfrac{\Delta_T-\Delta_S}{2}$, $\delta_g = -\sqrt{\dfrac{2}{3}}\dfrac{b_1}{m_Q}$ and the mass parameters Δ_S and Δ_T which represent the mass splittings between positive and negative parity doublets. They can be expressed in terms of the spin-averaged masses: $\Delta_S = \overline{M}_S - \overline{M}_H$ and $\Delta_T = \overline{M}_T - \overline{M}_H$ with

$$\begin{aligned}
\overline{M}_H &= \frac{3M_{P^*}+M_P}{4} \\
\overline{M}_S &= \frac{3M_{P_1'}+M_{P_0^*}}{4} \\
\overline{M}_T &= \frac{5M_{P_2^*}+3M_{P_1}}{8}\ .
\end{aligned} \tag{21}$$

The parameters in the various terms of the effective Lagrangian are universal and their determination is important in the definition of the effective theory and in the applications to the hadron phenomenology. Data recently collected on charmed and charmed-strange mesons, together with information on previously known positive parity charmed states, allow us to determine some of them. We identify $D_{sJ}(2317)$ and $D_{sJ}(2460)$ with the members of the $J^P_{s_\ell}(0^+,1^+)_{1/2}$ doublet and, using with the masses of the other charmed states reported in the PDG [3], we obtain the values of λ_H, λ_S and λ_T reported in Table 4 [23].

Table 4: λ_i parameters obtained using data in PDG [3]. The spin-averaged masses for the various doublets and the mass splittings Δ_S and Δ_T are also reported.

	$c\bar{q}$	$c\bar{s}$
λ_H	$(261.1\pm0.7\ \mathrm{MeV})^2$	$(270.8\pm0.8\ \mathrm{MeV})^2$
λ_S	$(265\pm57\ \mathrm{MeV})^2$	$(291\pm2\ \mathrm{MeV})^2$
λ_T	$(259\pm10\ \mathrm{MeV})^2$	$(266\pm6\ \mathrm{MeV})^2$
$\overline{M}_H$	$1974.8\pm0.4\,\mathrm{MeV}$	$2076.1\pm0.5\,\mathrm{MeV}$
$\overline{M}_S$	$2397\pm28\,\mathrm{MeV}$	$2424\pm1\,\mathrm{MeV}$
$\overline{M}_T$	$2445.1\pm1.4\,\mathrm{MeV}$	$2558\pm1\,\mathrm{MeV}$
Δ_S	$422\pm28\,\mathrm{MeV}$	$348\pm1\,\mathrm{MeV}$
Δ_T	$470.3\pm1.5\,\mathrm{MeV}$	$482\pm1\,\mathrm{MeV}$

In the above determinations we have neglected the mixing angle between the two 1^+ states D_1 and D_1'. Considering, instead, the result $\theta_c = -0.10\pm0.03\pm0.02\pm0.02\ rad$ [29] and using Δ_T and Δ_S in Table 4 together with eq. (20) and $m_c = 1.35$ GeV, we can compute the coupling b_1 in (17):

$$b_1 = 0.008\pm0.006\ \mathrm{GeV}^2\ , \tag{22}$$

therefore compatible with zero.

To determine the couplings h' and f in eqs. (6)-(16) we consider the widths of the two members of the $c\bar{q}\ s_\ell^P = \frac{3}{2}^+$ doublet, D_1 and D_2^* together with recent results from Belle Collaboration [29]:

$$\begin{aligned}
\Gamma(D_2^{*0}) &= 45.6\pm4.4\pm6.5\pm1.6\ \mathrm{MeV} \\
\Gamma(D_1^0) &= 23.7\pm2.7\pm0.2\pm4.0\ \mathrm{MeV}\ .
\end{aligned} \tag{23}$$

In the plane (h', f) four regions are allowed by data which, due to symmetry $(h', f)\to(-h', -f)$, reduce to the two inequivalent regions depicted in fig. 3.

A further constraint is the Belle measurement of the helicity angle distribution in the decay $D_{s1}(2536)\to D^{*+}K_S^0$, with the determination of the ratio

$$R = \frac{\Gamma_s}{\Gamma_s+\Gamma_d}\ , \tag{24}$$

$\Gamma_{s,d}$ being the s and d wave partial widths, respectively [30]: $0.277\le R\le 0.955$ (a measurement of the ratio R versus the phase difference between s and d was obtained by CLEO Collaboration for non-strange mesons [31]). Although the range of R is wide, it allows to exclude the region B in fig.3, leaving only the region A that can be represented as

$$h' = 0.45\pm0.05 \qquad f = 0.044\pm0.044\ \mathrm{GeV}\ . \tag{25}$$

The coupling constant f is compatible with zero, hence the contribution of the lagrangian term (16) is small. Since also the coupling b_1 turns out to be small, the two 1^+ states corresponding to the $s_\ell^P = \frac{1}{2}^+, \frac{3}{2}^+$ practically coincide with the physical states. For the width of $D_{s1}(2536)$ we predict

$$\Gamma(D_{s1}(2536)) = 2.5\pm1.6\ \mathrm{MeV} \tag{26}$$

compatible with the present bound: $\Gamma(D_{s1}(2536)) < 2.3$ MeV [3].

Conclusions

Our knowledge of charm spectroscopy has greatly improved in recent years, but there are results which challenge our understanding of some aspects of Quantum Chromodynamics. The question whether the many newly observed states are conventional or exotic ones has been put forward in several cases, stimulating numerous investigations. We have discussed the most recently observed $c\bar{s}$ mesons, adopting a classification in terms of doublets provided in the heavy quark limit.

Our conclusion is represented by table 5, where we propose identification of the states discussed above. In the table, $D_{sJ}(2317)$ and $D_{sJ}(2460)$ are identified as the

Table 5: $c\bar{s}$ states organized according to s_ℓ^P and J^P. The mass of known mesons is indicated. States in bold face have been placed in the table according to the interpretation supported in this paper.

s_ℓ^P	$\frac{1}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^-$
$n=1$					
$J^P = s_\ell^P - \frac{1}{2}$	$D_s(1965)$ (0^-)	$D_{sJ}(2317)$ (0^+)	$D_{s1}(2536)$ (1^+)	(1^-)	(2^-)
$J^P = s_\ell^P + \frac{1}{2}$	$D_s^*(2112)$ (1^-)	$D_{sJ}(2460)$ (1^+)	$D_{s2}^*(2573)$ (2^+)	(2^-)	$\mathbf{D_{sJ}(2860)}$ (3^-)
$n=2$					
$J^P = s_\ell^P - \frac{1}{2}$	(0^-)	(0^+)	(1^+)	(1^-)	(2^-)
$J^P = s_\ell^P + \frac{1}{2}$	$\mathbf{D_{sJ}(2710)}$ (1^-)	(1^+)	(2^+)	(2^-)	(3^-)

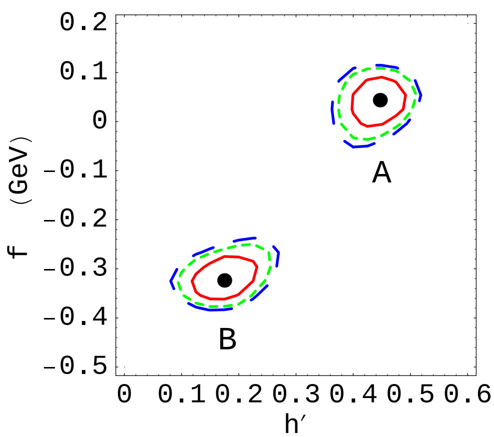

Figure 3: Regions in the (h', f) plane constrained by the widths of D_2^{*0} and D_1^0. Only the region A is also compatible with the constraints on the parameter R in eq.(24).

two members of the $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$ doublet. As for the other two states, it is likely that the interpretation of $D_{sJ}(2710)$ as the first radial excitation of D_s^* is correct. The identification of $D_{sJ}(2860)$ is still under debate.

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